Digital Communication Systems
ECS 452

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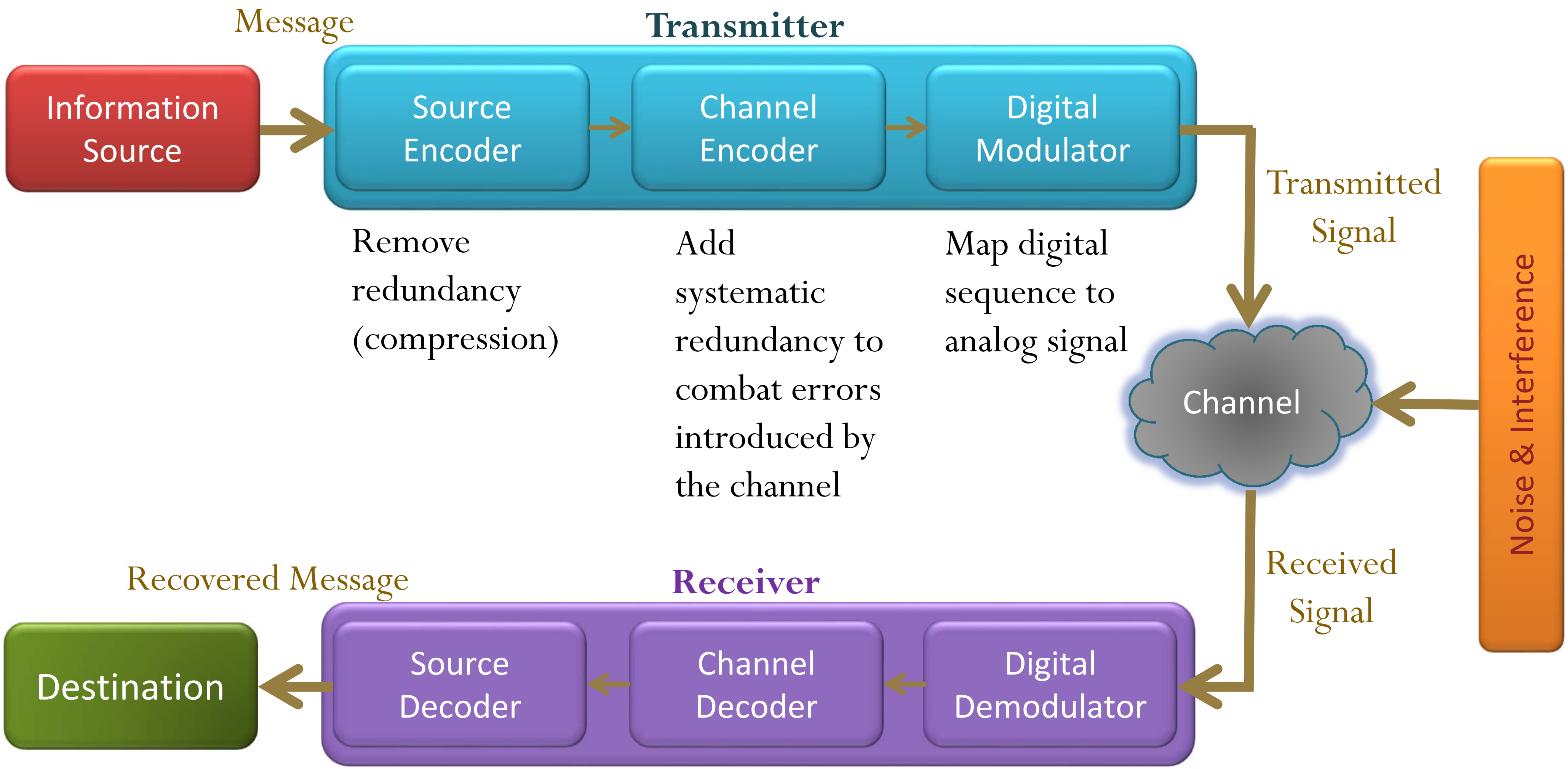
6. Digital Modulation

Office Hours:

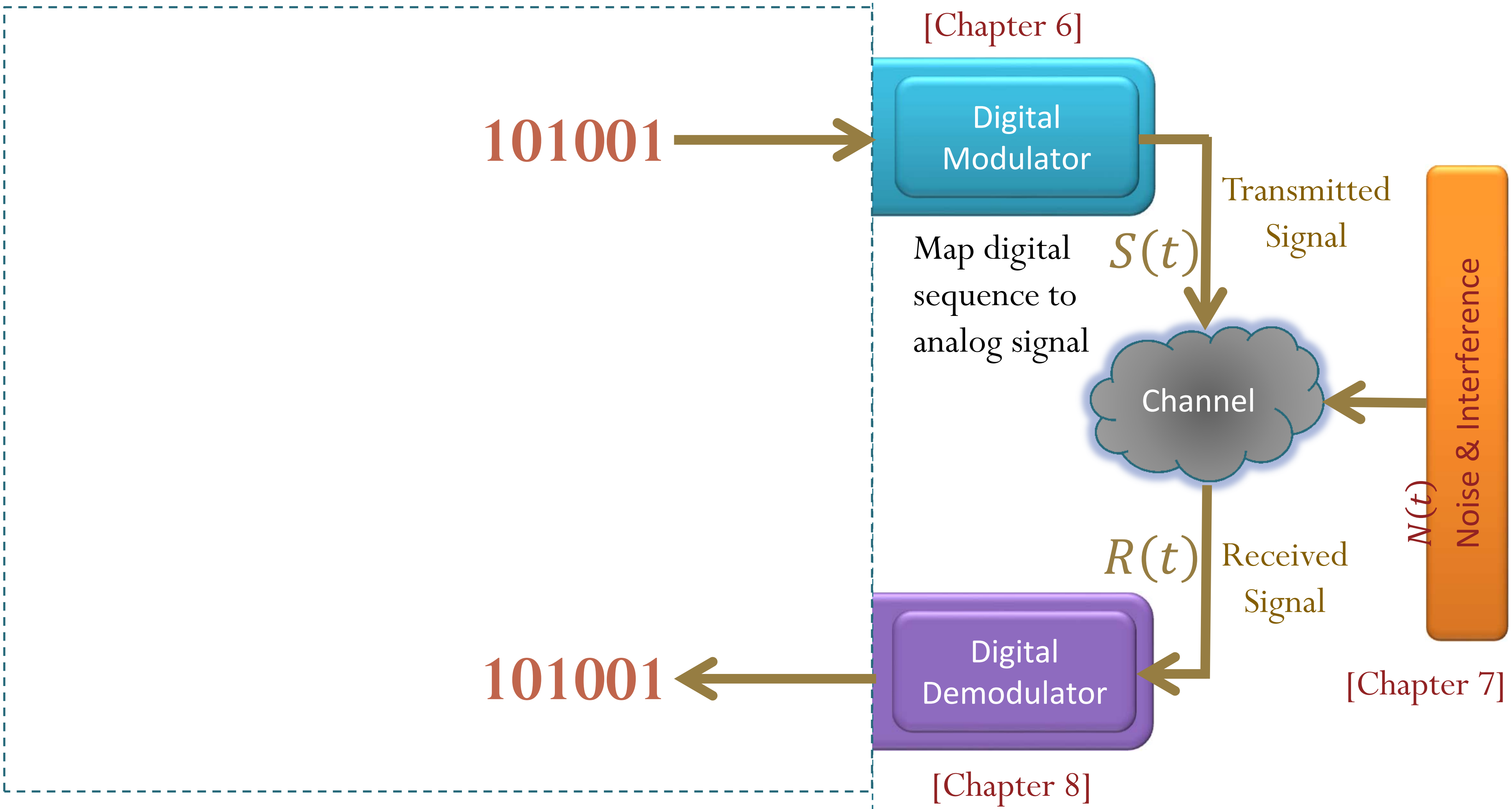
BKD, 6th floor of Sirindhralai building

Monday	10:00-10:40
Tuesday	12:00-12:40
Thursday	14:20-15:30

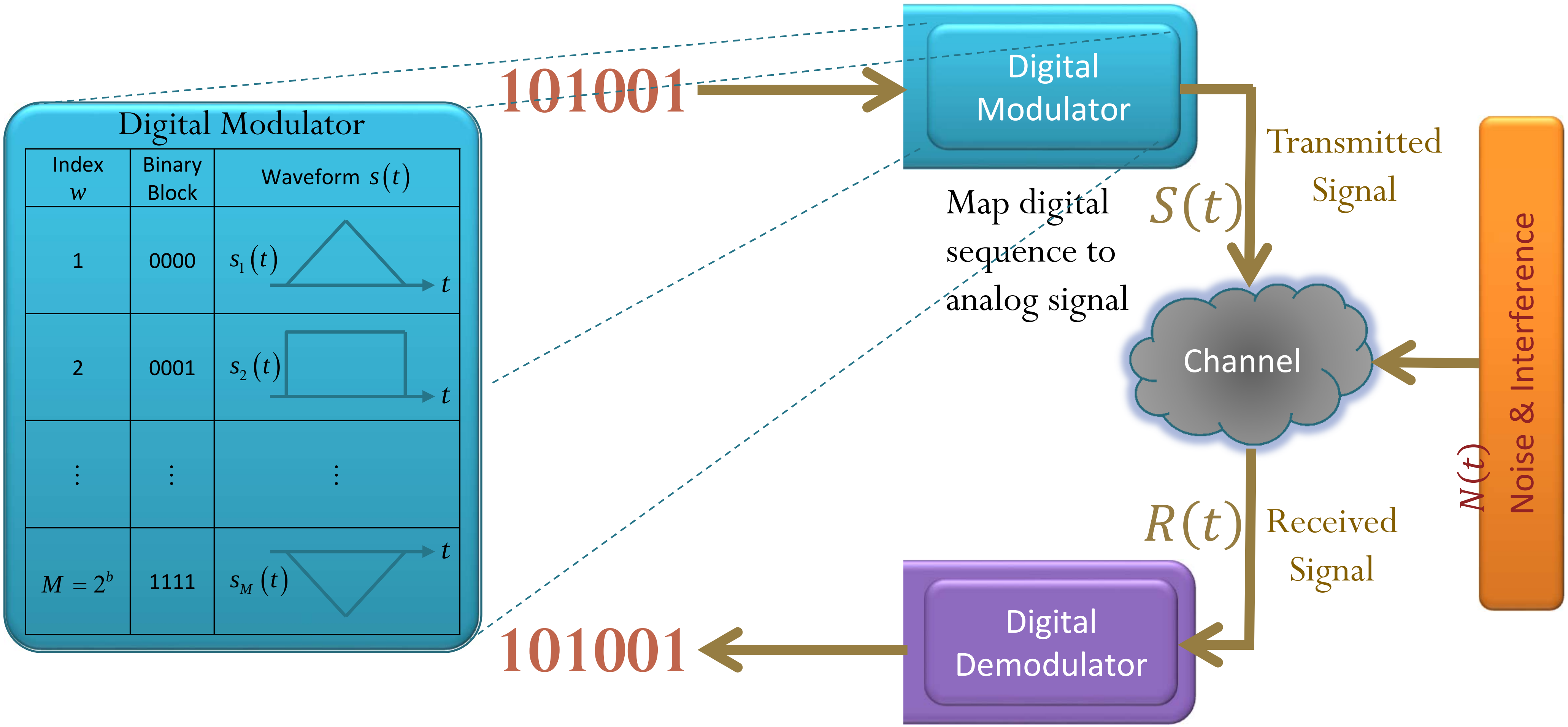
Elements of digital commu. sys.



Digital Modulation/Demodulation



Digital Modulation/Demodulation



Energy and Power

- Consider a signal $g(t)$.
- Total (normalized) **energy**:

Parseval's Theorem

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} |g(t)|^2 dt \doteq \int_{-\infty}^{\infty} |G(f)|^2 df.$$

$$\Psi_g(f) = |G(f)|^2$$

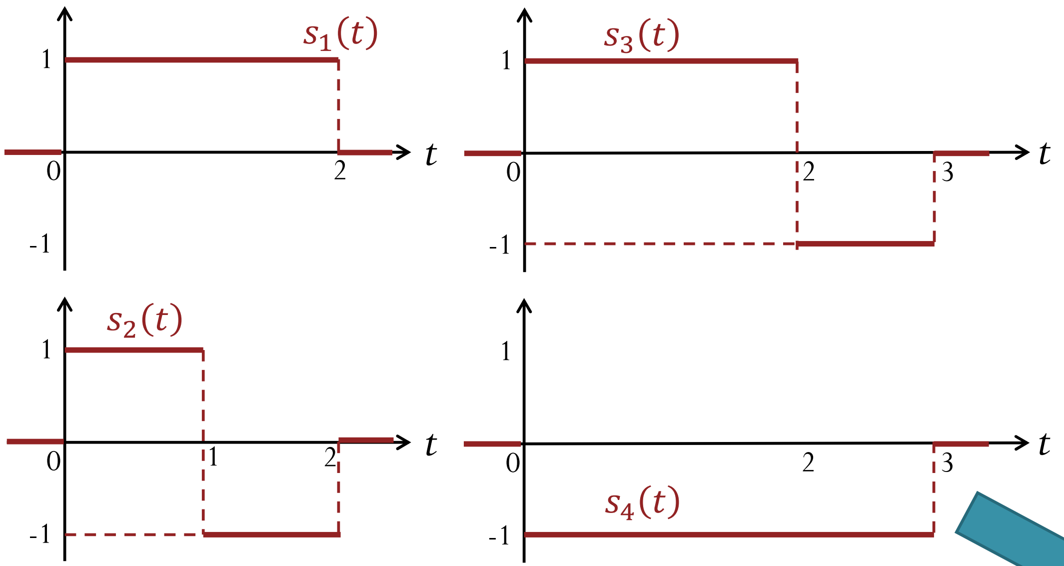
ESD: Energy Spectral Density

- Average (normalized) **power**:

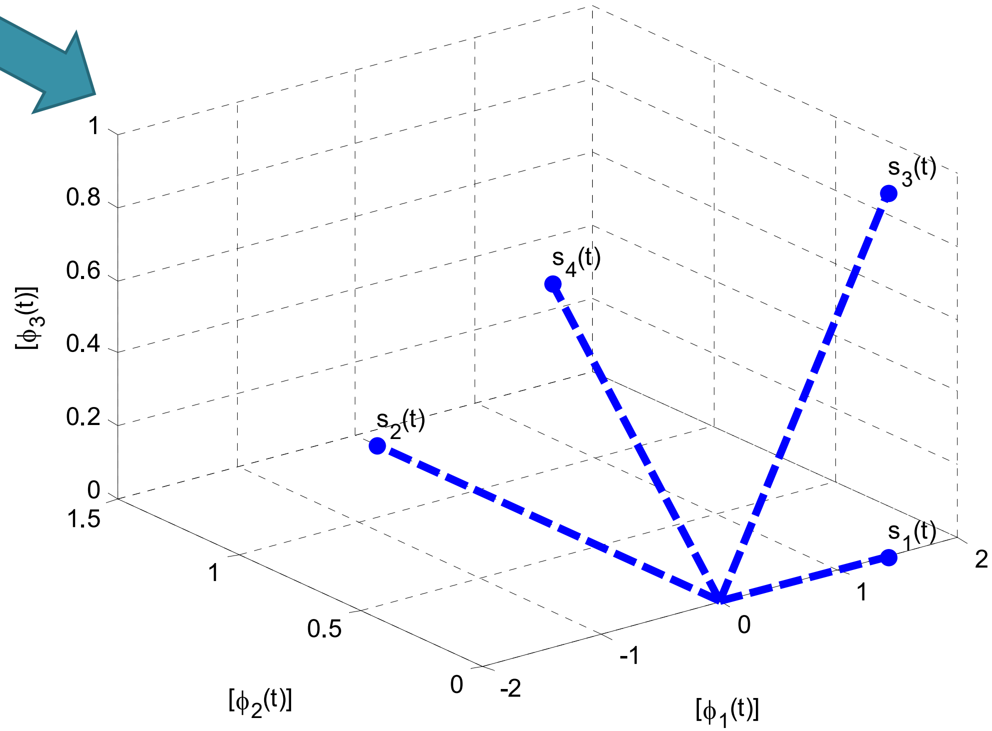
$$P_g = \left\langle |g(t)|^2 \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 dt.$$

time-average operator

Review: From Waveforms to Constellation



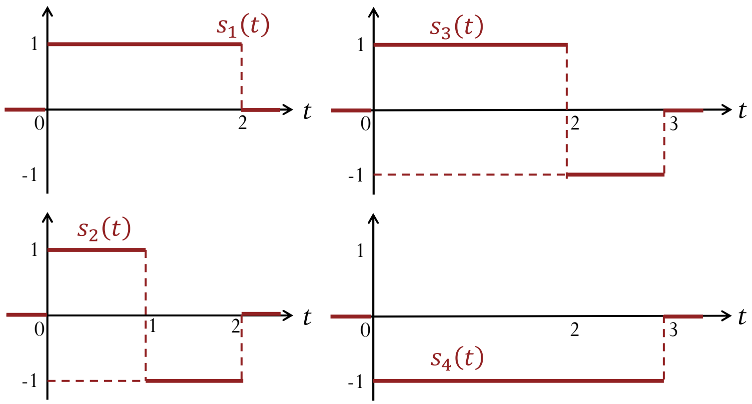
- What should be the axes?
  - 1 axis = 1 dimension
  - How many do we need?
- How to represent the waveform on these axes?

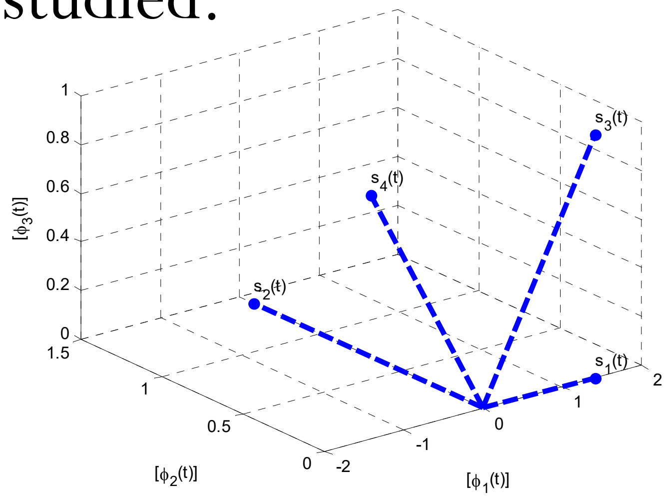


- A waveform contains infinitely many points. To represent *all* possible waveforms, we would need to work in infinite-dimensional space.
- However, we only have to consider four possible waveforms here.

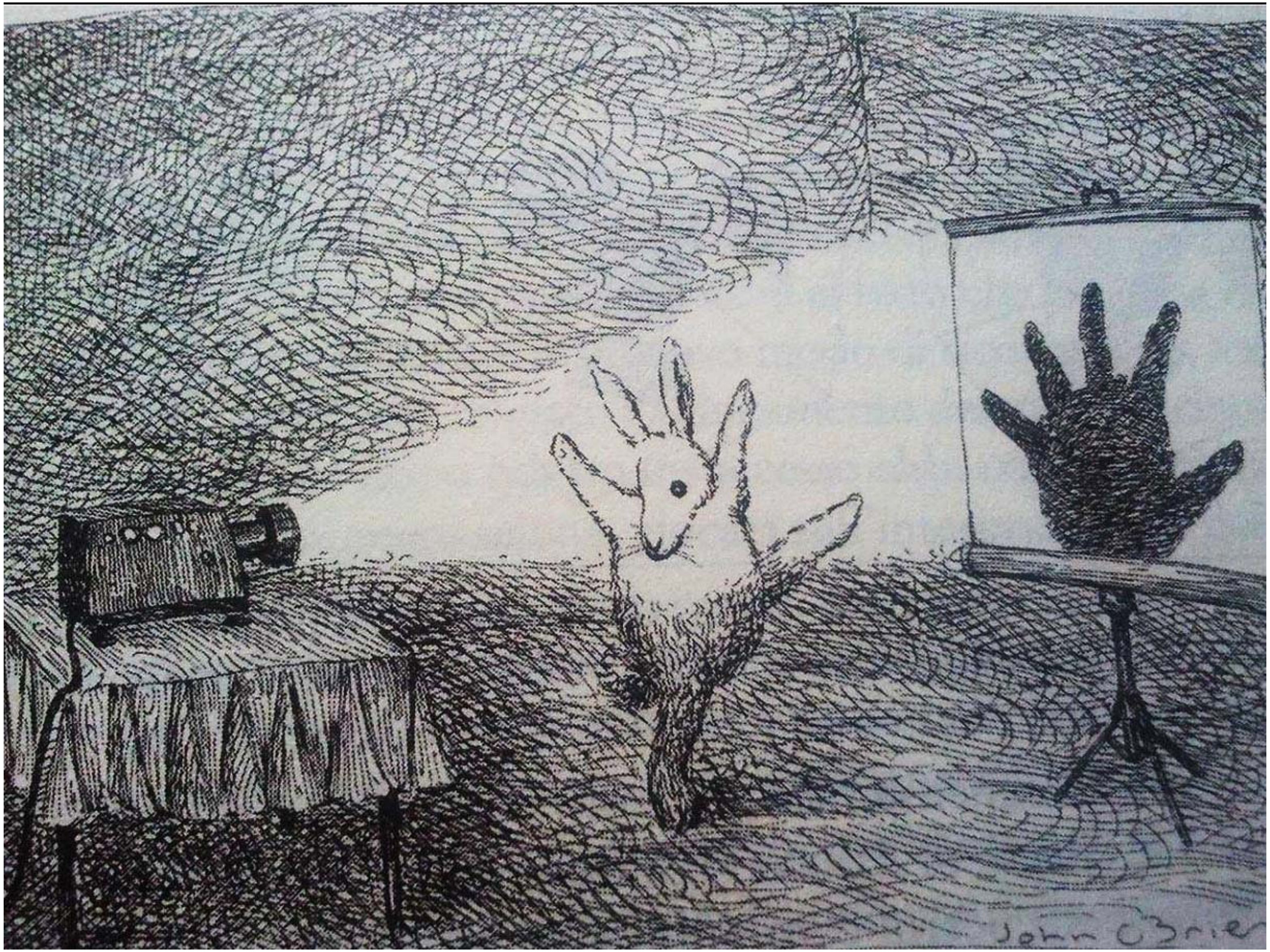
Review: From Waveforms to Constellation

- In Section 6.2, we start with a collection of vectors instead of a collection of waveforms.
  - The calculation for vector case is easier.
  - It may seem redundant because some implicit axes are already there when we specify any vector.
    - Still useful because the calculation can remove the redundant axes.
  - The formula is exactly the same for the waveform case.
- In Section 6.3, the waveform case is studied.

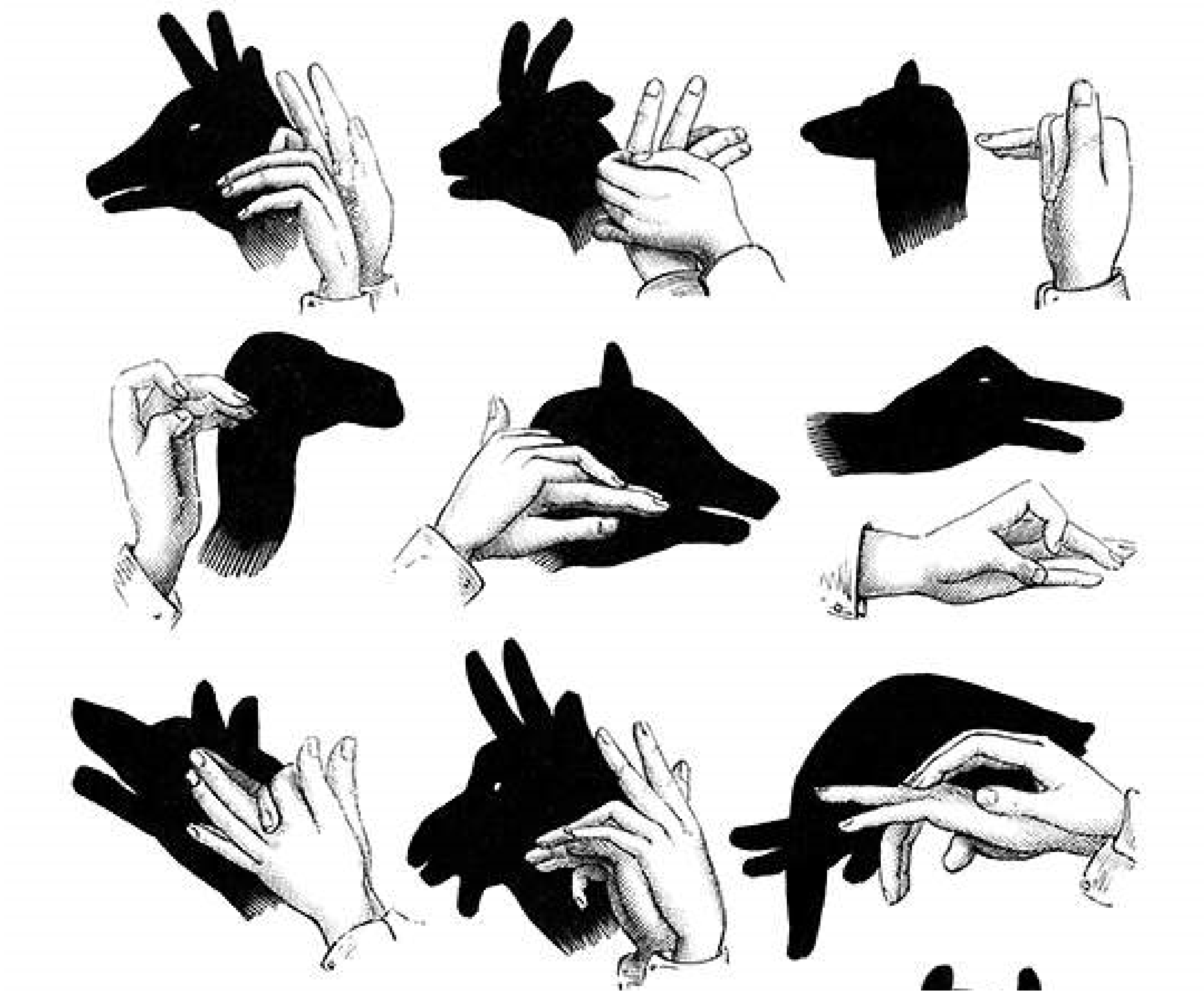




Projection



Projection: Hand Shadow



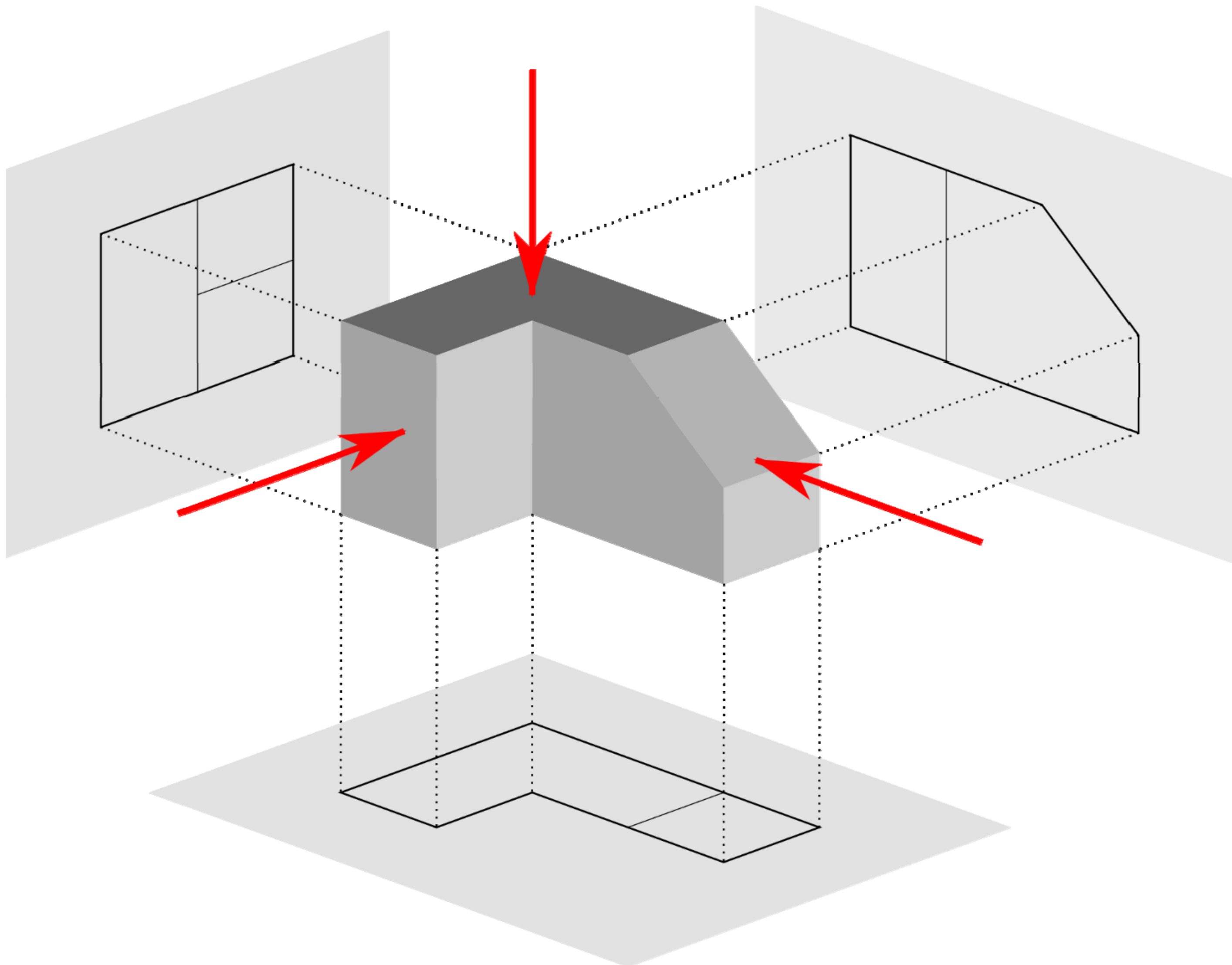
Projection: Shadow Art

[http://thesuiteworld.com/wp-content/uploads/2011/07/Shadows-thesuiteworld.jpg]

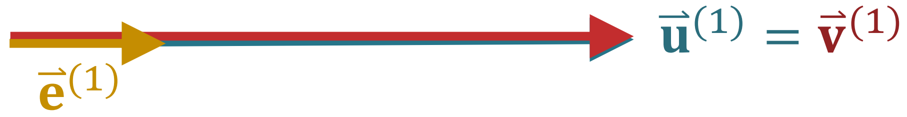
Projection: Shadow Art

[https://www.youtube.com/watch?v=VJp6wbHk02s]

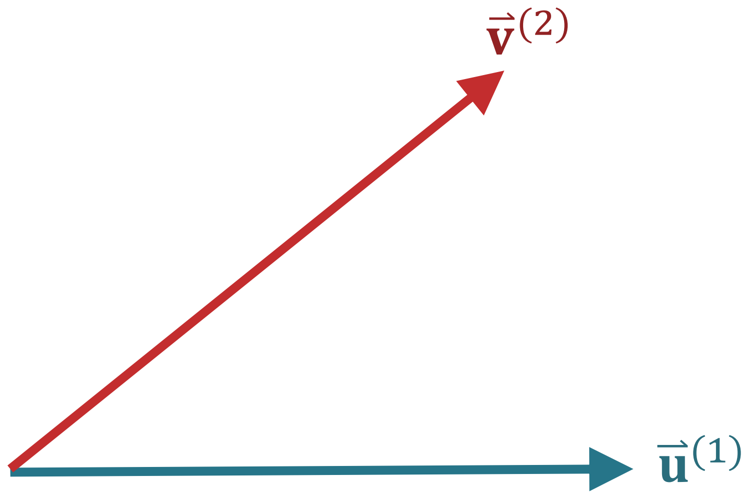
Projection: Engineering drawing



GSOP

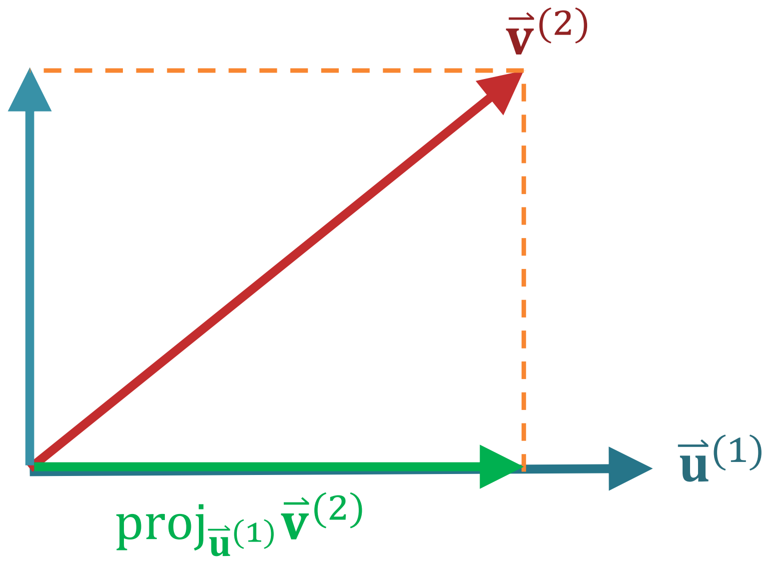


GSOP

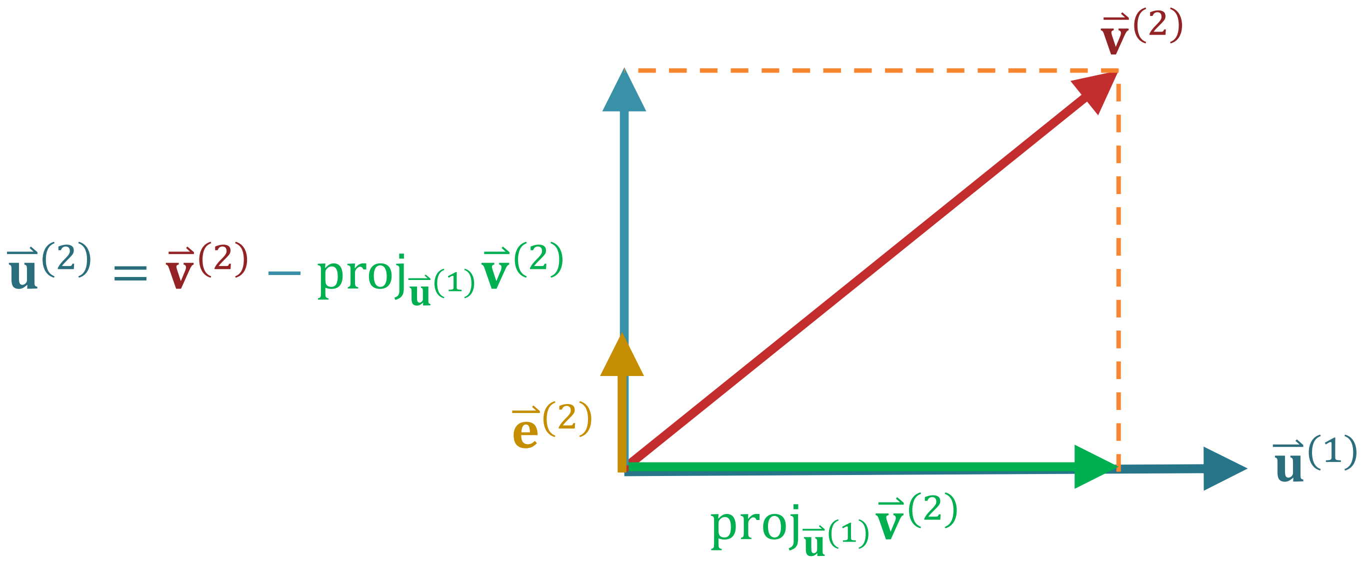


GSOP

$$\vec{\mathbf{u}}^{(2)} = \vec{\mathbf{v}}^{(2)} - \text{proj}_{\vec{\mathbf{u}}^{(1)}} \vec{\mathbf{v}}^{(2)}$$

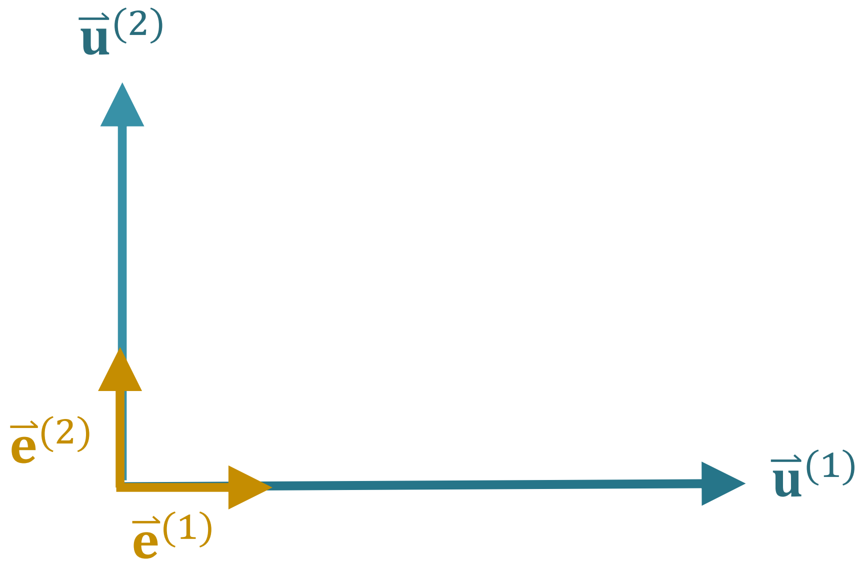


GSOP

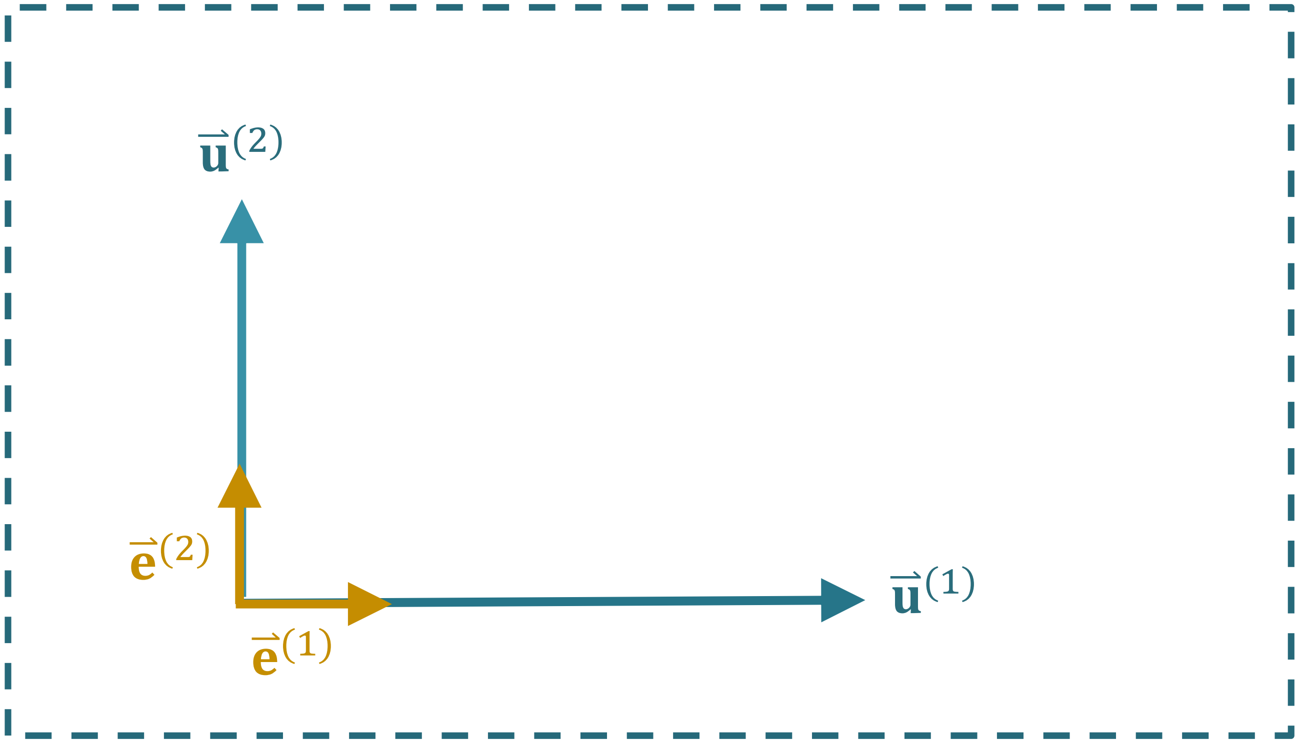


$\vec{\mathbf{u}}^{(2)} = \vec{\mathbf{v}}^{(2)} - \mathrm{proj}_{\vec{\mathbf{u}}^{(1)}} \vec{\mathbf{v}}^{(2)}$
$\vec{\mathbf{v}}^{(2)}$
$\vec{\mathbf{e}}^{(2)}$
$\vec{\mathbf{u}}^{(1)}$
$\mathrm{proj}_{\vec{\mathbf{u}}^{(1)}} \vec{\mathbf{v}}^{(2)}$

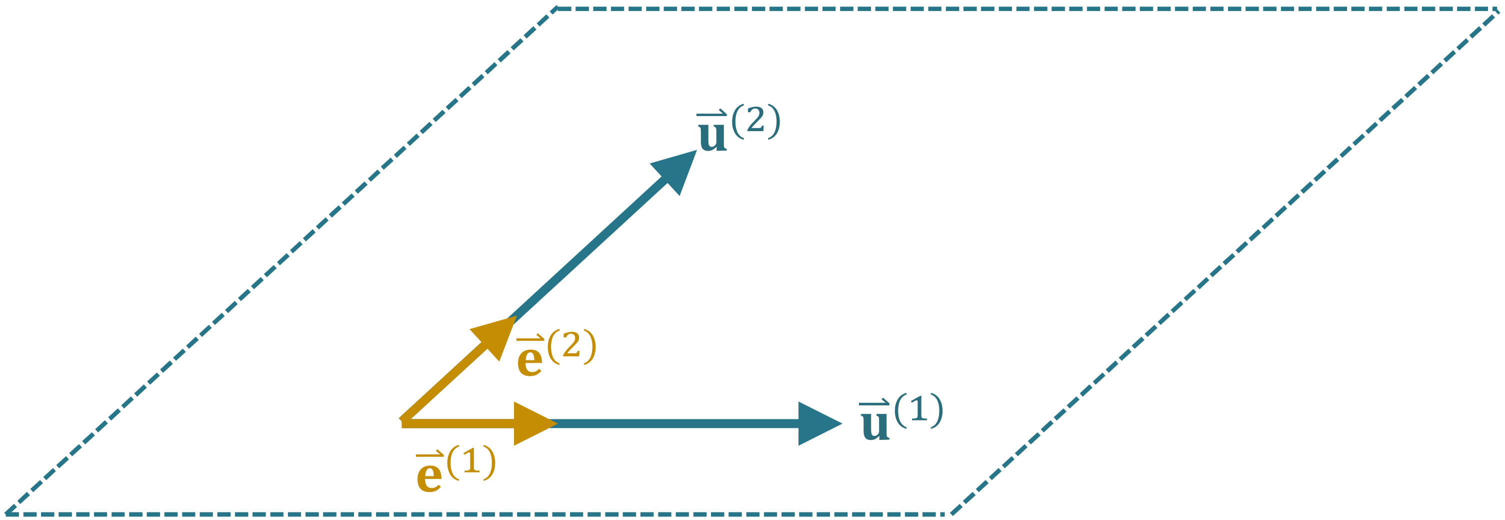
GSOP



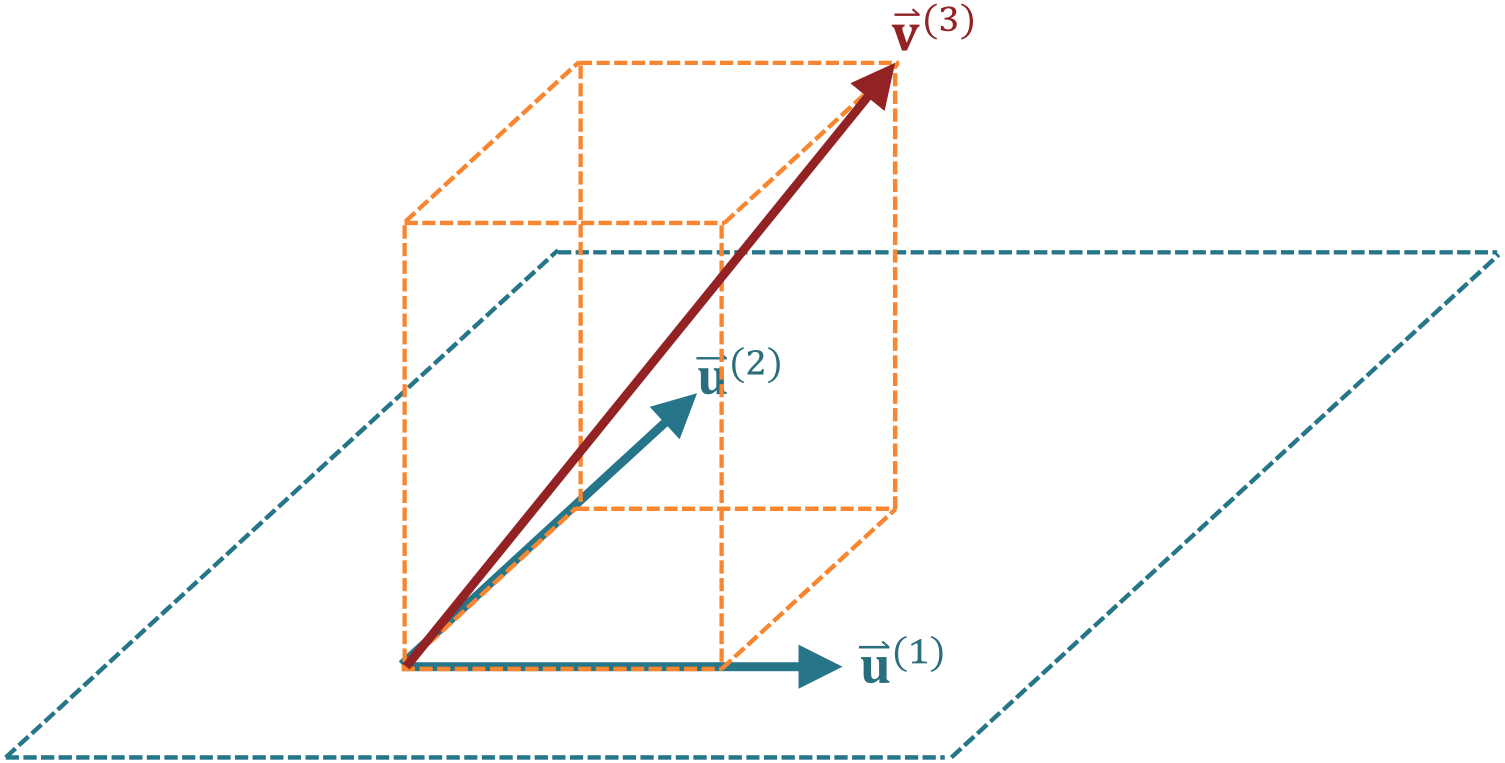
GSOP



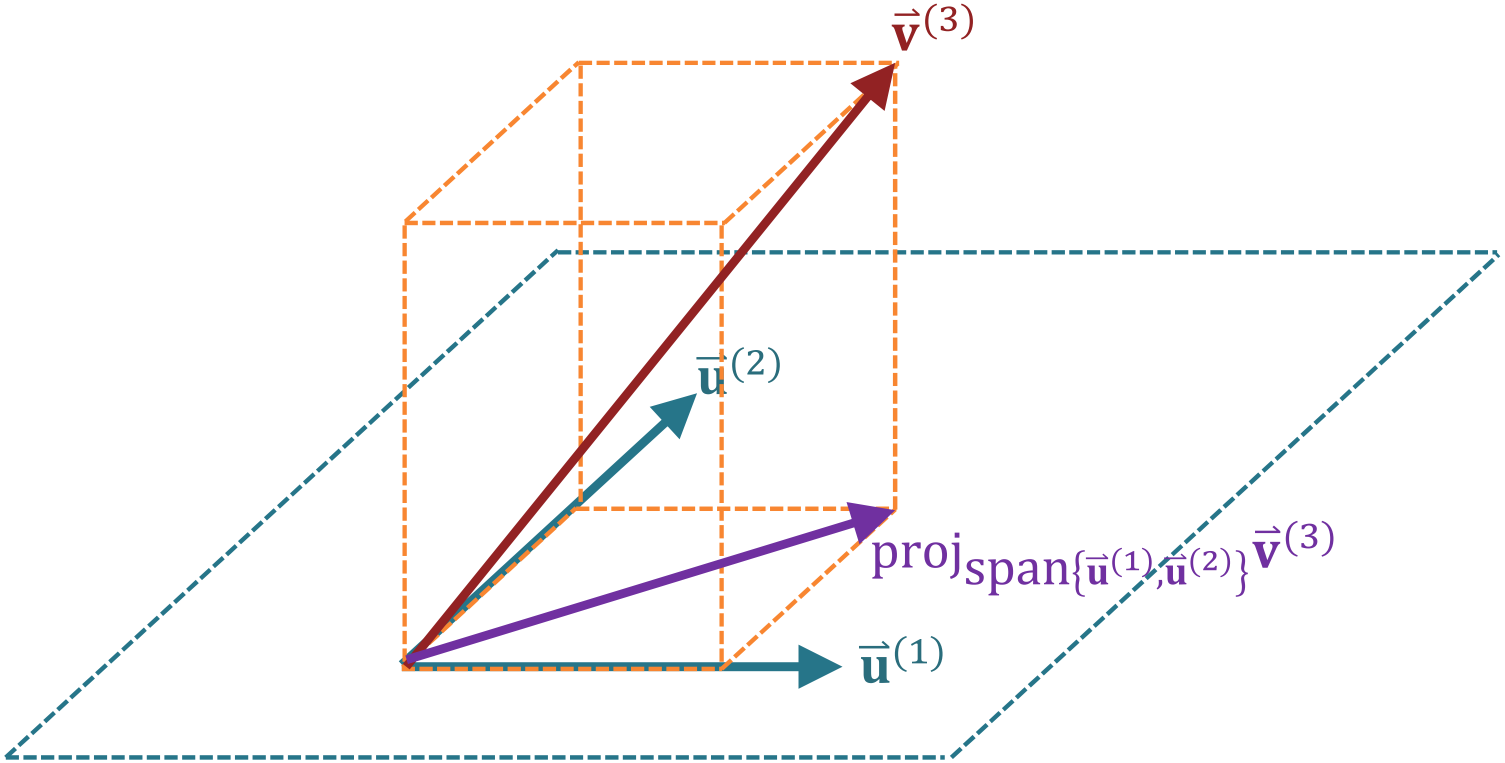
GSOP



GSOP

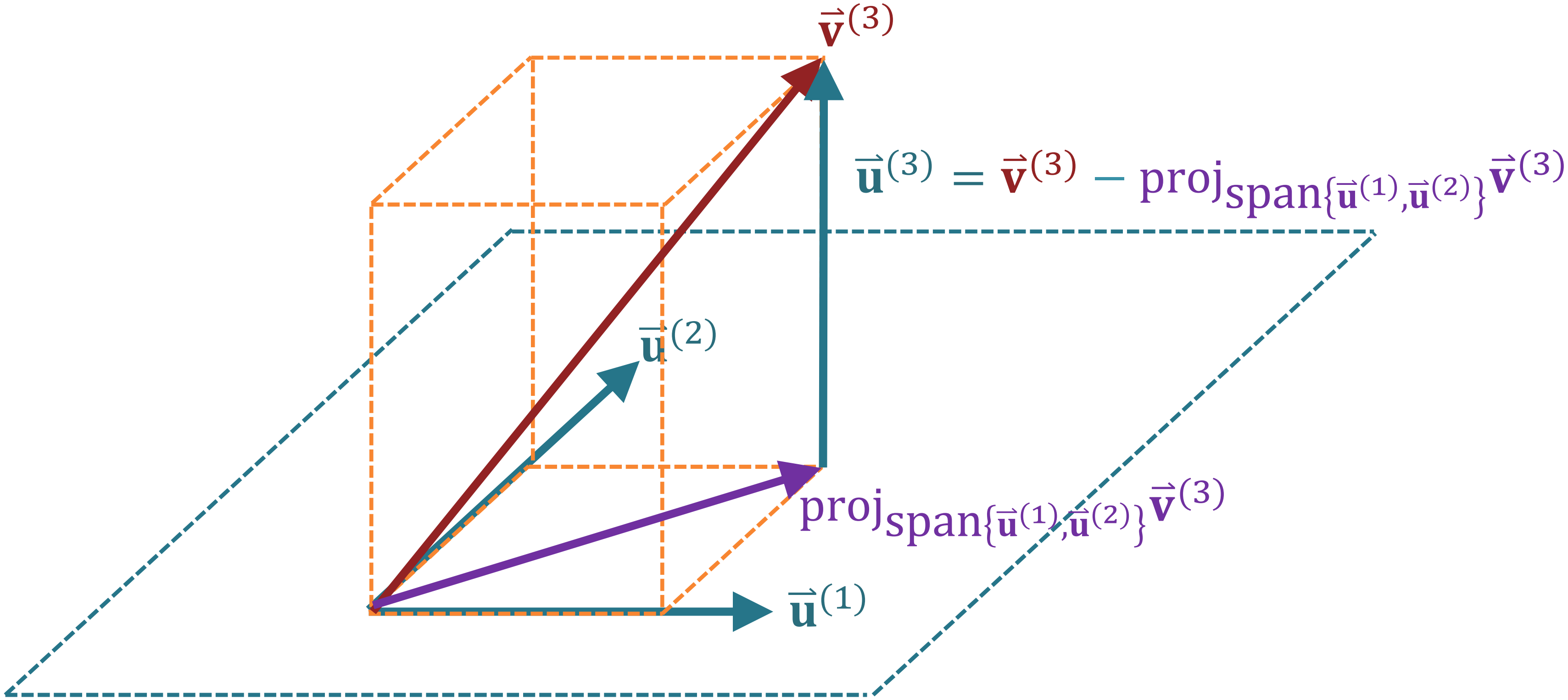


GSOP



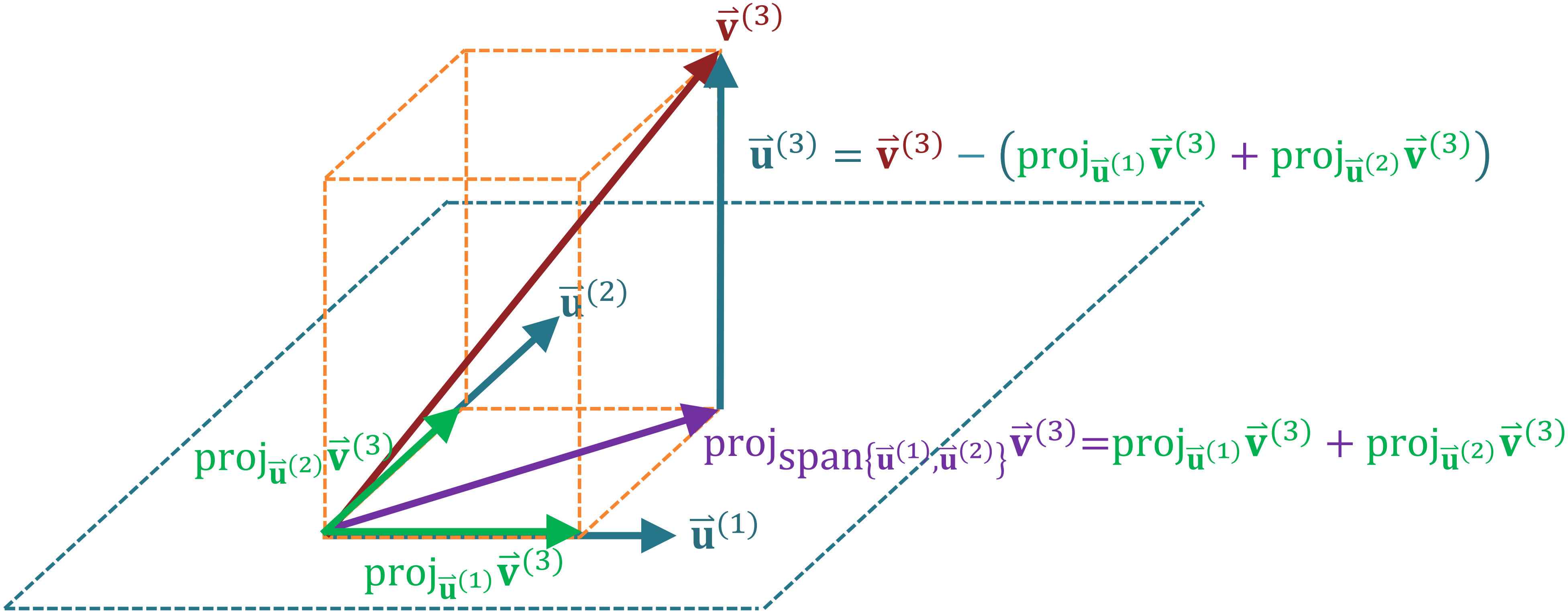
$\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(2)}$
$\text{proj}_{\text{span}\{\vec{\mathbf{u}}^{(1)},\vec{\mathbf{u}}^{(2)}\}}\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(1)}$

GSOP

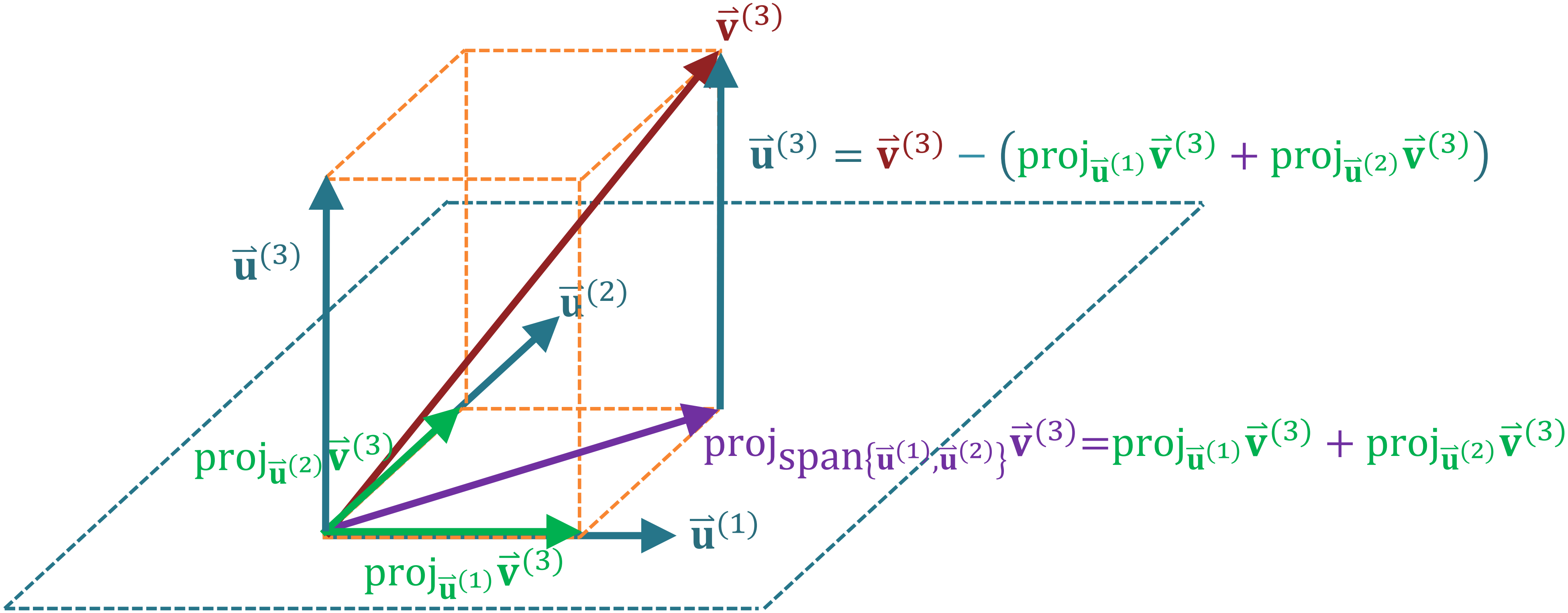


$\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(3)} = \vec{\mathbf{v}}^{(3)} - \text{proj}_{\text{span}\{\vec{\mathbf{u}}^{(1)},\vec{\mathbf{u}}^{(2)}\}}\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(2)}$
$\text{proj}_{\text{span}\{\vec{\mathbf{u}}^{(1)},\vec{\mathbf{u}}^{(2)}\}}\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(1)}$

GSOP

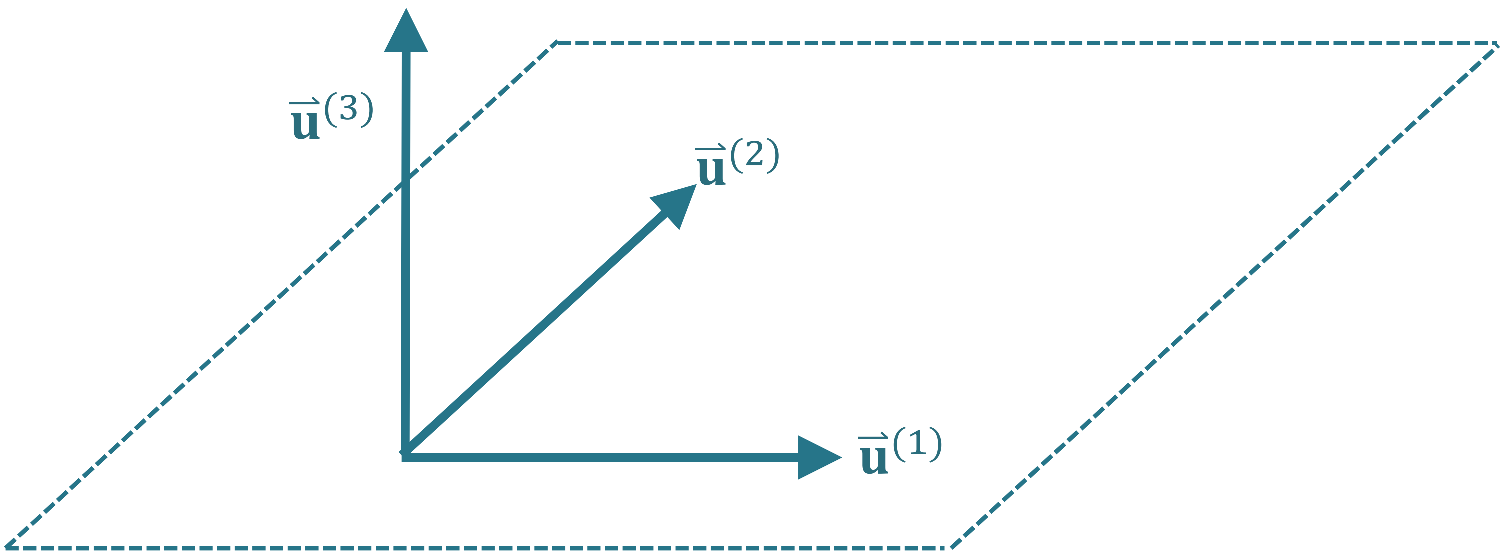

$\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(3)} = \vec{\mathbf{v}}^{(3)} - \left(\text{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)} + \text{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}\right)$
$\vec{\mathbf{u}}^{(2)}$
$\text{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}$
$\text{proj}_{\text{span}\{\vec{\mathbf{u}}^{(1)},\vec{\mathbf{u}}^{(2)}\}}\vec{\mathbf{v}}^{(3)} = \text{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)} + \text{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(1)}$
$\text{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)}$

GSOP

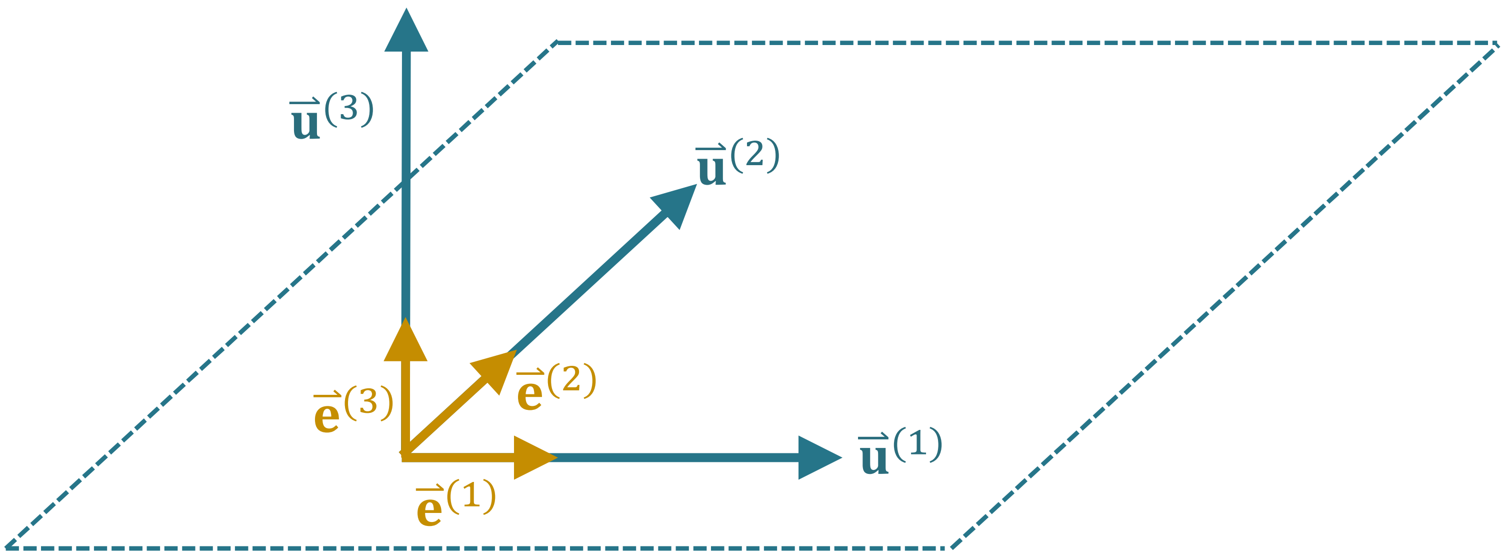


$\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(3)} = \vec{\mathbf{v}}^{(3)} - \left(\mathrm{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)} + \mathrm{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}\right)$
$\vec{\mathbf{u}}^{(3)}$
$\vec{\mathbf{u}}^{(2)}$
$\mathrm{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}$
$\mathrm{proj}_{\mathrm{span}\{\vec{\mathbf{u}}^{(1)},\vec{\mathbf{u}}^{(2)}\}}\vec{\mathbf{v}}^{(3)} = \mathrm{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)} + \mathrm{proj}_{\vec{\mathbf{u}}^{(2)}}\vec{\mathbf{v}}^{(3)}$
$\vec{\mathbf{u}}^{(1)}$
$\mathrm{proj}_{\vec{\mathbf{u}}^{(1)}}\vec{\mathbf{v}}^{(3)}$

GSOP



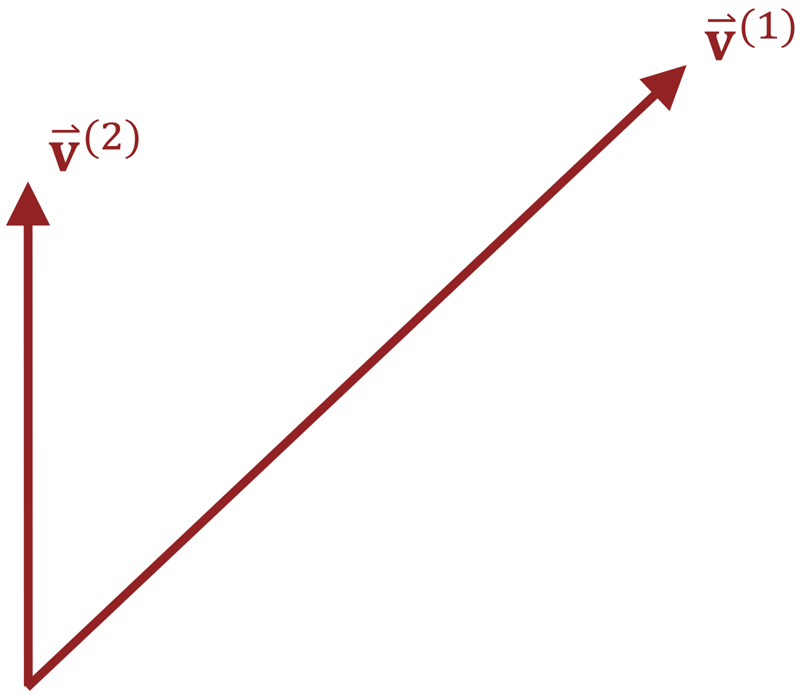
GSOP



$\vec{\mathbf{u}}^{(3)}$
$\vec{\mathbf{u}}^{(2)}$
$\vec{\mathbf{e}}^{(2)}$
$\vec{\mathbf{e}}^{(3)}$
$\vec{\mathbf{u}}^{(1)}$
$\vec{\mathbf{e}}^{(1)}$

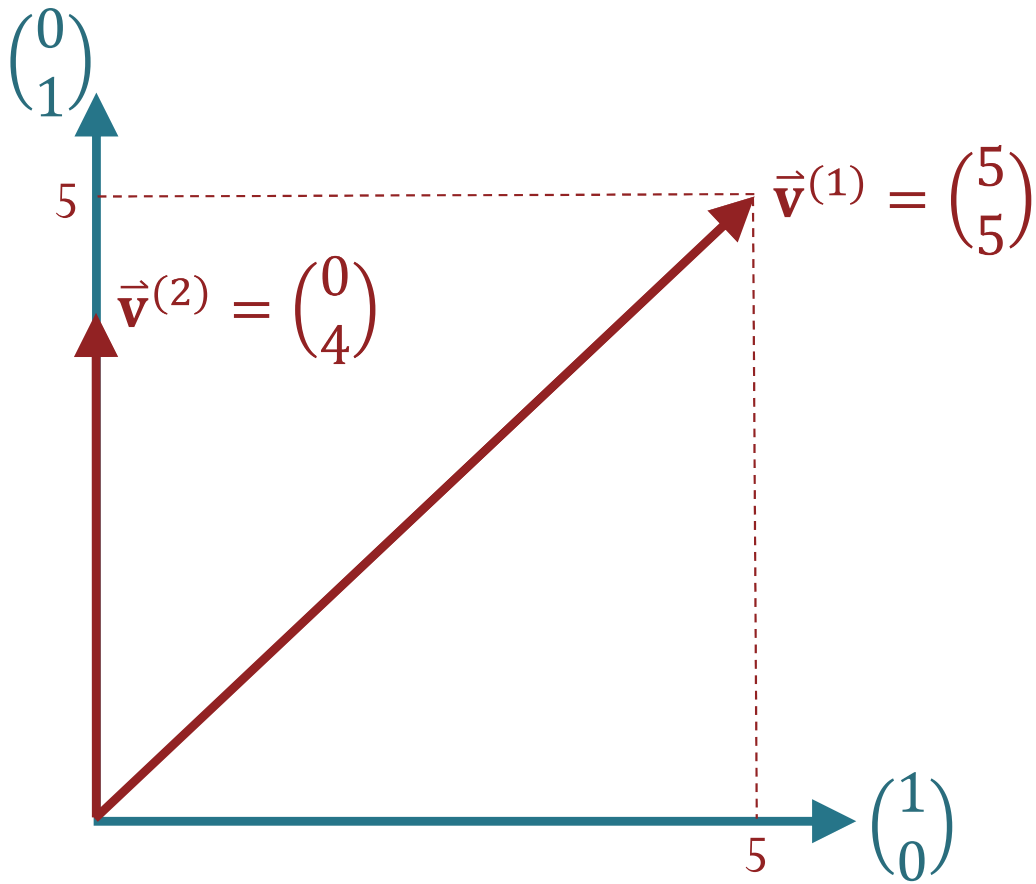
Changing basis / coordinate systems

Two vectors: $\vec{\mathbf{v}}^{(1)}$ and $\vec{\mathbf{v}}^{(2)}$



Changing basis / coordinate systems

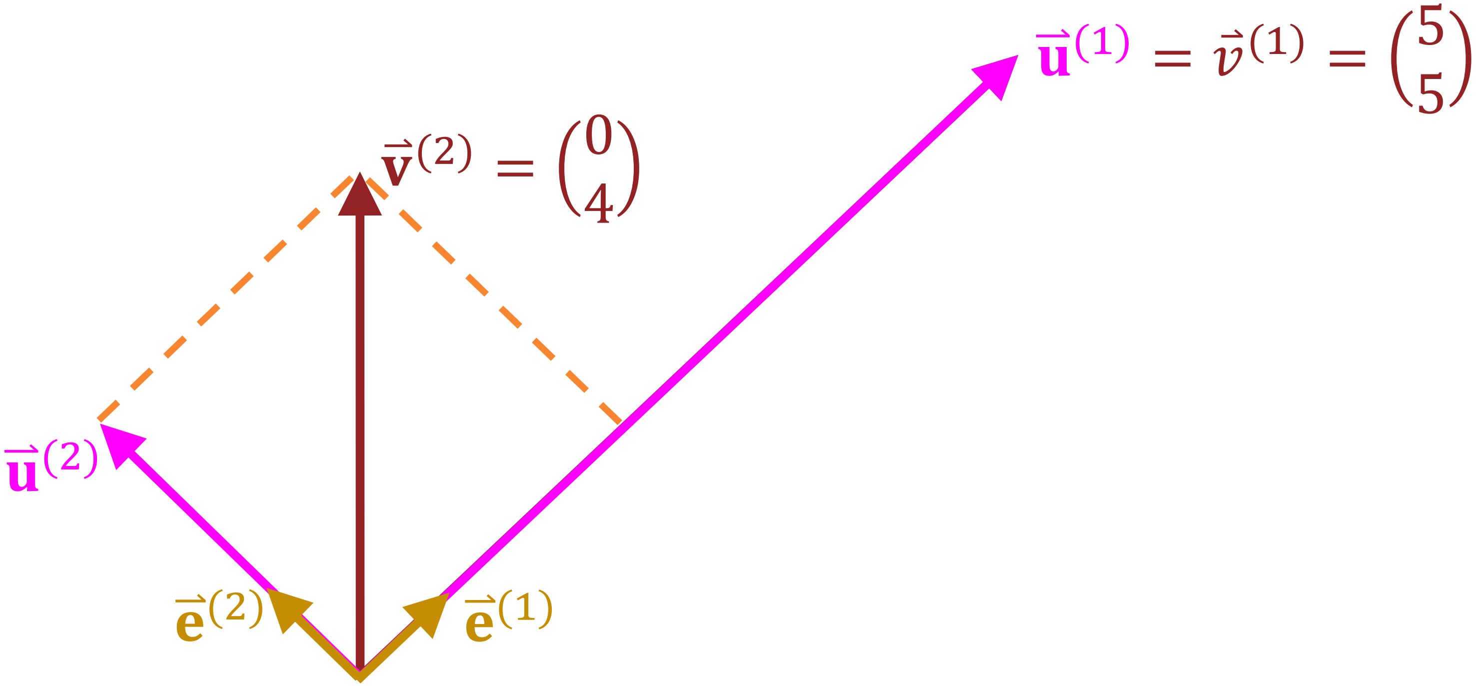
Old Coordinates: $\vec{\mathbf{v}}^{(1)} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and $\vec{\mathbf{v}}^{(2)} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$



Changing basis / coordinate systems

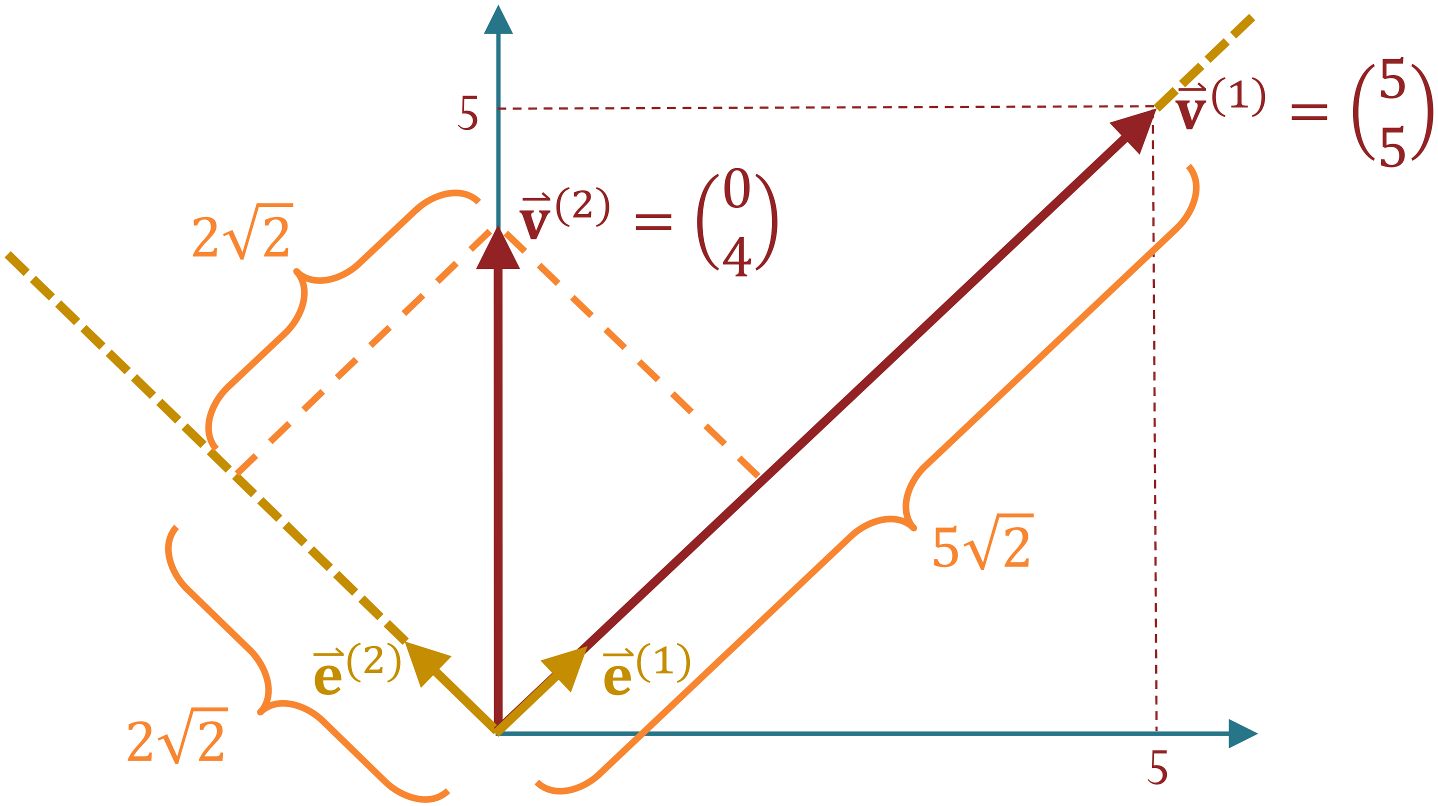
Old Coordinates: $\vec{\mathbf{v}}^{(1)} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ and $\vec{\mathbf{v}}^{(2)} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

Applying GSOP:



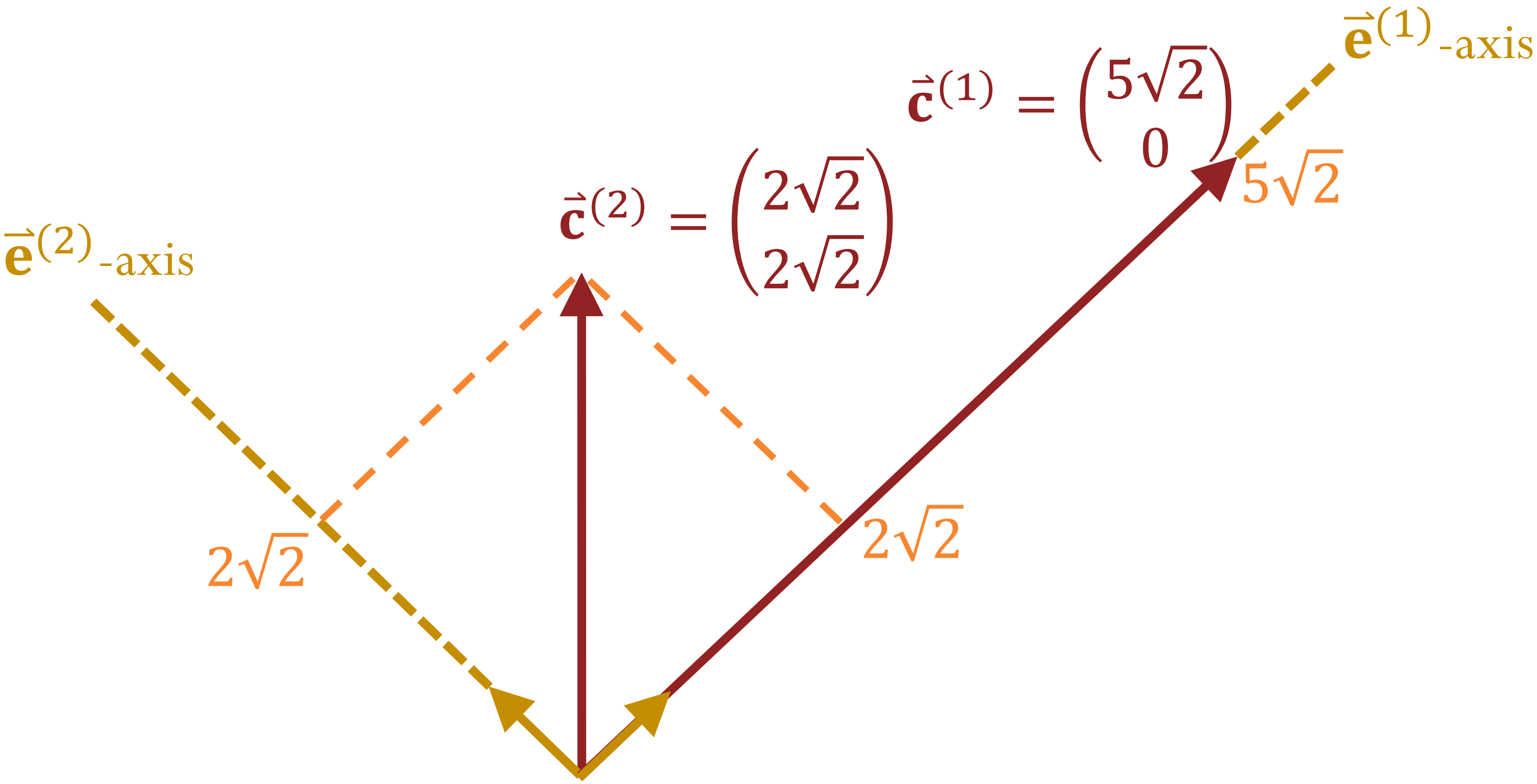
Changing basis / coordinate systems

New axes: $\vec{\mathbf{e}}^{(1)}$ and $\vec{\mathbf{e}}^{(1)}$

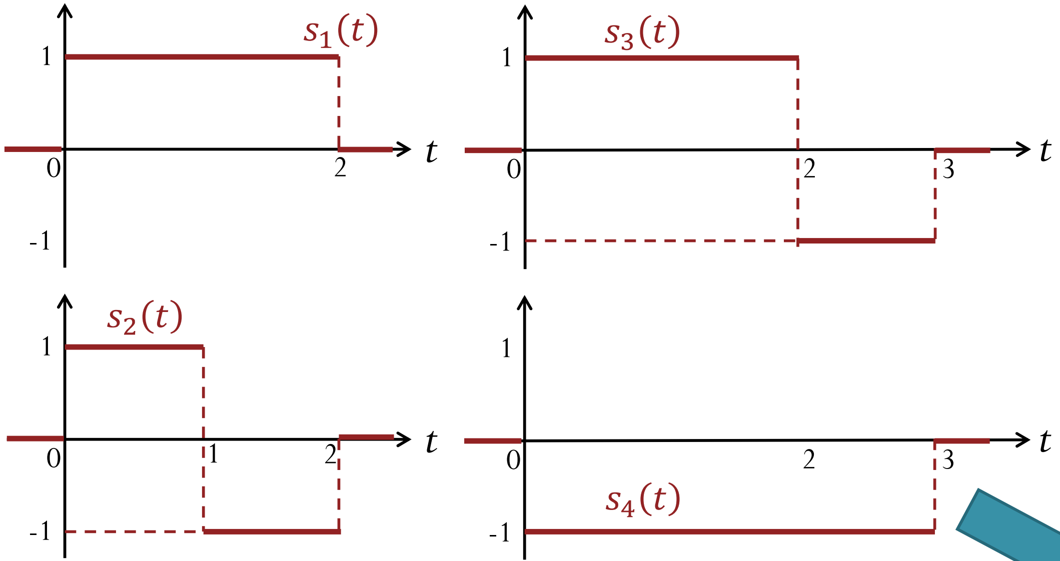


Changing basis / coordinate systems

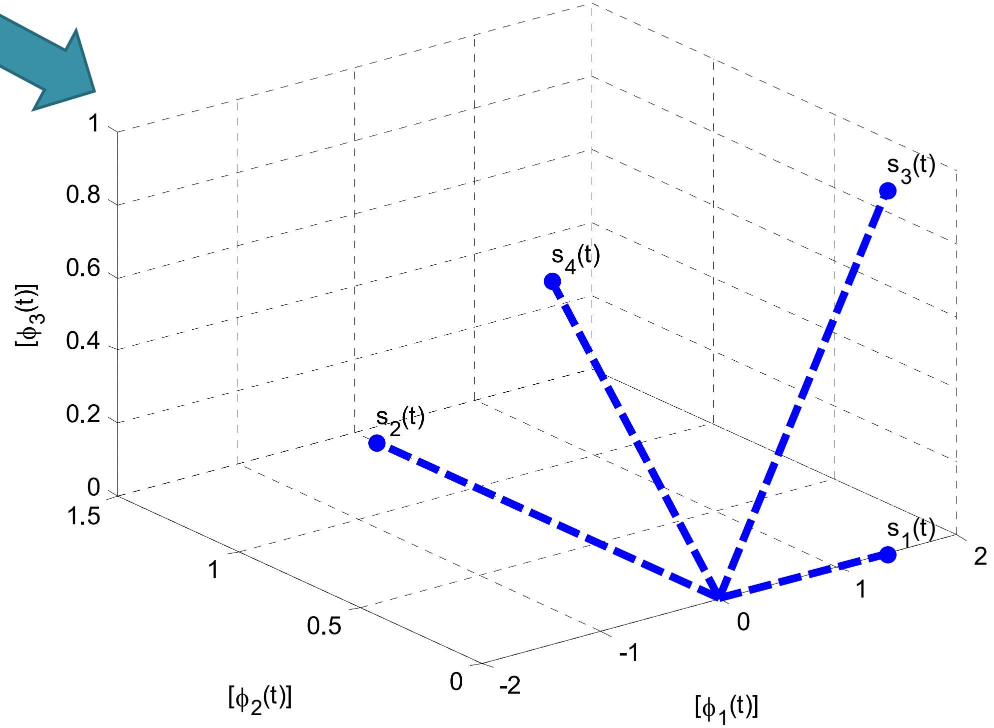
New Coordinates: $\vec{\mathbf{c}}^{(1)} = \begin{pmatrix} 5\sqrt{2} \\ 0 \end{pmatrix}$ and $\vec{\mathbf{c}}^{(2)} = \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}$



From Waveforms to Constellation



Use GSOP to find N orthonormal basis functions $\{\phi_1(t), \phi_2(t), \dots, \phi_K(t)\}$ to use as axes for $\{s_1(t), s_2(t), \dots, s_M(t)\}$.

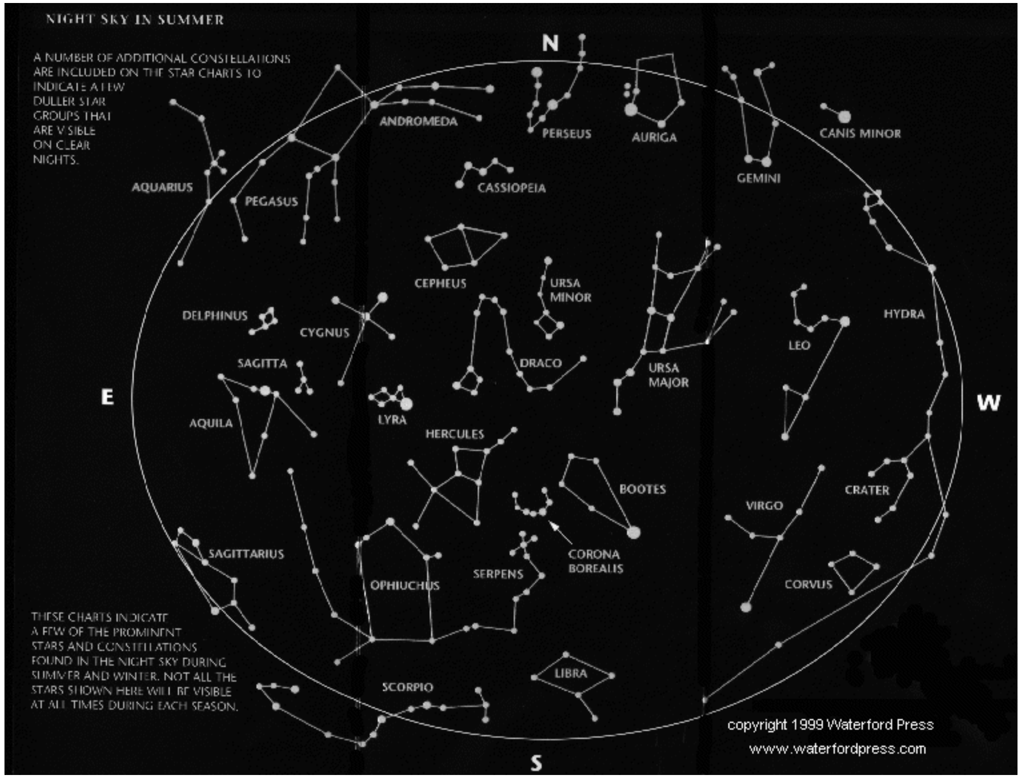


This gives vector representations for the waveforms $s_1(t), s_2(t), \dots, s_M(t)$:

$$\vec{\mathbf{s}}^{(1)}, \vec{\mathbf{s}}^{(2)}, \dots, \vec{\mathbf{s}}^{(M)}$$

which can be visualized in the form of signal constellation

Star Constellations



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[http://68.media.tumblr.com/89eed4669ec511bbb6413acb8da8b0e2/tumblr_mz3qmyQeLH1rhb9f5o1_r1_400.jpg]

Review: QAM

4.8 Quadrature Amplitude Modulation (QAM)

Definition 4.82. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband real-valued signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the corresponding QAM signal:

Form #1 $$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

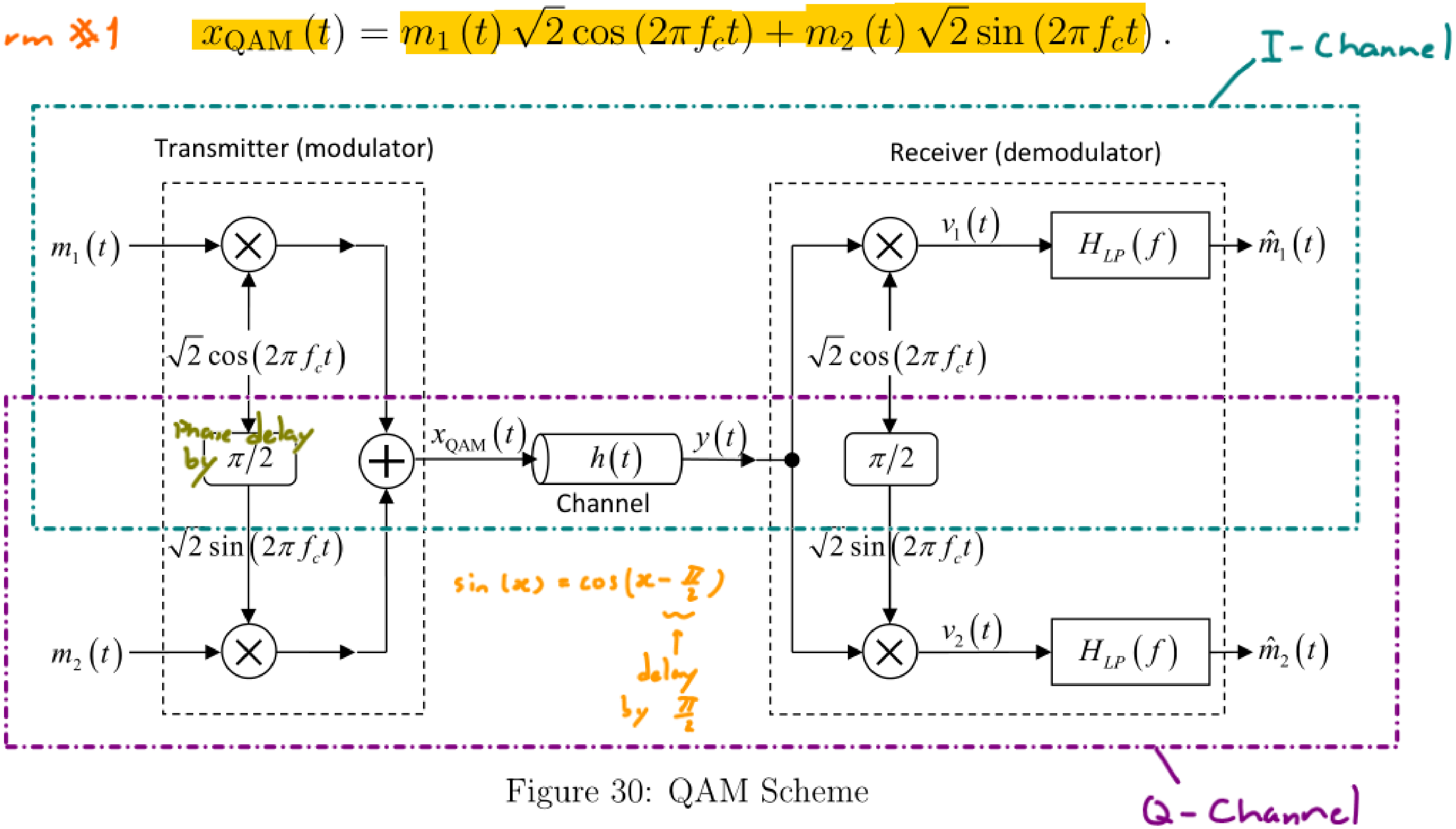

Figure 30: QAM Scheme

Review: QAM

4.87. Sinusoidal form (envelope-and-phase description [3, p. 165]):

calculator

Form # 2

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where

$$\text{envelope:} \quad E(t) = |m_1(t) - jm_2(t)| = \sqrt{m_1^2(t) + m_2^2(t)}$$

$$\text{phase:} \quad \phi(t) = \angle\left(m_1(t) - jm_2(t)\right)$$

4.90. Complex form:

$$x_{\text{QAM}}(t) = \sqrt{2}\text{Re}\left\{(m(t))\, e^{j2\pi f_c t}\right\}$$

where[20] $m(t) = m_1(t) - jm_2(t)$.

- We refer to $m(t)$ as the ***complex envelope*** (or ***complex baseband signal***) and the signals $m_1(t)$ and $m_2(t)$ are known as the ***in-phase*** and ***quadrature(-phase)*** components of $x_{\text{QAM}}(t)$.

Standard Quaternary QAM